PROBLEMS.

58. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.

Two men, A and B, in Boston, hire a carriage for $25, to go to Concord, N. H., and back, the distance being 72 miles, with the privilege of taking in three more persons. Having gone 20 miles, they take in C; at Concord, they take in D; and when within 30 miles of Boston, they take in E. How much shall each man pay? [From *Greenleaf's National Arithmetic.*]

59. Proposed by ISAAC L. BEVERAGE, Monterey, Virginia.

A broker charges me $1\frac{1}{2}$ per cent. brokerage for buying some uncurrent bank bills at 20 per cent. discount. Of these bills 4 of $50. each become worthless, but the remainder I dispose of at par, and make by the operation $364. What was the face amount? [Which answer is correct, $3000, or $$3048\frac{24}{37}$?]

ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

56. Proposed by CHAS. E. MYERS, Canton, Ohio, and Hon. JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

(a) How much can be paid for a bond, bearing 5 per cent. interest, and having ten years to run, so as to realize 3 per cent. on the investment? (b) At what price must the government sell 5 per cent. $100 bonds to run ten years, interest payable annually, to make them the same to the buyer as 3 per cent. bonds at par, to run ten years, interest payable annually, provided the buyer can invest all interest received at 4 per cent. interest, payable annually?

Solution by J. K. ELWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania.

Let x=price, a=face, n=number of periods, R=rate bond bears, r=rate to be realized, r'=rate on interest.

The interest on bond is an annuity at compound interest whose final value $=\frac{Ra}{r'}[(1+r')^n-1]$, which added to the *face* value of bond must equal the compound amount of the *price* for n periods, or $x(1+r)^n$.

$\therefore x=\dfrac{a+Ra[(1+r')^n-1]}{(1+r)^n}$. For ($a$), $a=100$, $n=10$, $R=.05$, $r=.03$, $r'=.03$.

$$\therefore\ x=\frac{100+\ \frac{5}{.03}(1.03^{10}-1)}{1.03^{10}}=\$117.0604.$$

For (b), $a=100$, $n=10$, $R=.05$, $r=.03$, $r'=.04$.

$$\therefore\ x=\frac{100+\ \frac{5}{.04}(1.04^{10}-1)}{1.03^{10}}=\$119.0777.$$

If in (a) interest were payable semi-annually, we should have $a=100$, $n=20$, $R=.025$, $r=.015$, $r'=.015$, and $x=\$117.168+$, or \$117.17 as given in the tables of bond values used by brokers and bankers.

Also solved by *E. W. MORRELL, B. F. YANCEY* and *G. B. M, ZERR*. Prof. Morrell obtained as results \$118.856 and \$117.661; and Proposer, to last part, \$117.60.

57. Proposed by J. C. CORBIN, Pine Bluff, Arkansas.

Find the quotient of

$$\begin{vmatrix} (s-a_1)^2 & a_1^2 & a_1^2\ldots\ldots a_1^2 \\ a_2^2 & (s-a_2)^2 & a_2^2\ldots\ldots a_2^2 \\ a_3^2 & a_3^2 & (s-a_3)^2\ldots\ldots a_3^2 \\ \ldots & \ldots & \ldots \\ a_n^2 & a_n^2 & a_n^2\ldots s-a_n^2 \end{vmatrix} \div \begin{vmatrix} s-a_1 & a_1 & a_1\ldots\ldots\ldots a_1 \\ a_2 & s-a_2 & a_2\ldots\ldots\ldots a_2 \\ a_3 & a_3 & s-a_3\ldots\ldots\ldots a_3 \\ \ldots & \ldots & \ldots \\ a_n & a_n & a_n\ldots\ldots s-a_n \end{vmatrix}$$

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science in Texarkana College, Texarkana, Arkansas-Texas.

Let Q=the quotient and as we can exchange row for column without altering the value, we get

$$Q=\begin{vmatrix} (s-a_1)^2 & a_2^2 & a_3^2\ldots a_n^2 \\ a_1^2 & (s-a_2)^2 & a_3^2\ldots a_n^2 \\ a_1^2 & a_2^2 & (s-a_3)^2\ldots a_n^2 \\ \ldots & \ldots & \ldots \\ a_1^2 & a_2^2 & a_3^2.(s-a_n)^2 \end{vmatrix} \div \begin{vmatrix} s-a_1 & a_2 & a_3\ldots\ldots a_n \\ a_1 & s-a_2 & a_3\ldots\ldots a_n \\ a_1 & a_2 & s-a_3\ldots\ldots a_n \\ \ldots & \ldots & \ldots \\ a_1 & a_2 & a_3\ldots s-a_n \end{vmatrix}$$

All the elements in the i^{th} column of the numerator being a_i^2, of the denominator a_i, except in the i^{th} row which is $(s-a_i)^2$ for numerator, and $s-a_i$ for denominator. Hence, we have

$$Q=\begin{vmatrix} 1, & 0, & 0, & 0,\ldots\ldots \\ 1, & (s-a_1)^2, & a_2^2, & a_3^2,\ldots \\ 1, & a_1^2, & (s-a_2^2,) & a_3^2,\ldots \\ 1, & a_1^2, & a_2^2, & (s-a_3)^2,\ldots \\ \ldots & \ldots & \ldots & \ldots \end{vmatrix} \div \begin{vmatrix} 1, & 0, & 0, & 0,\ldots\ldots \\ 1, & s-a_1, & a_2, & a_3,\ldots \\ 1, & a_1, & s-a_2, & a_3,\ldots \\ 1, & a_1, & a_2, & s-a_3,\ldots \\ \ldots & \ldots & \ldots & \ldots \end{vmatrix}$$

Multiply first column of numerator by a_i^2, of the denominator by a_i and subtract from the i^{th} column; do this for each column and the value is unaltered.